Twisted homological stability for configuration spaces

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Abstract:

The nth unordered configuration space $C_n(M)$ of a space M is the space of subsets of M of cardinality n, naturally topologised. If M is the interior of a manifold with non-empty boundary, there are natural maps $C_n(M) \longrightarrow C_{n+1}(M)$ that "push" a new point into M from the boundary. If M is connected, these maps have been shown by Arnol'd, Segal and McDuff to induce isomorphisms on homology in a certain range of degrees — in this sense, one says that configuration spaces are *homologically stable*.

I will present a result showing that configuration spaces are moreover *homologically stable for finite-degree twisted coefficient systems* (I will first explain in detail what these are), and explain the main ideas of the proof. Taking $M = \mathbb{C} = \mathbb{R}^2$, this includes as a special case the (classifying spaces of the) braid groups. I will then discuss some examples of finite-degree twisted coefficient systems for the braid groups that may be built from the homology of regular covers of configuration spaces on punctured surfaces, including the Burau and the Lawrence-Krammer-Bigelow representations (this last part is part of joint work in progress with Arthur Soulié).

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