Homological stability for moduli spaces of disconnected submanifolds

Martin Palmer-Anghel

Abstract

The story of homological stability for configuration spaces began with the work of V. I. Arnol'd in 1970, who proved that the homology of the braid groups B_n stabilises: in each degree it is independent of the number of strands n once n is sufficiently large (compared to the degree). The configuration space of n unordered particles in the plane is a classifying spaces for B_n, and, using this viewpoint, the result of Arnol'd was soon generalised by G. Segal and D. McDuff to configuration spaces of unordered particles in an arbitrary (connected, open) manifold. Moreover, they identified a space whose homology is isomorphic to the "stable homology" of the sequence of configuration spaces — for example, the relevant space for the braid groups is the double loopspace of the 3-sphere, $\Omega^2 S^3$.

I will present two generalisations of these stabilisation results, in which (a) point particles are replaced by more general closed submanifolds and (b) homology is considered with twisted coefficients, where the coefficients for each space in the sequence assemble into a finite-degree functor on a certain category.

In this talk I will sketch the ideas behind the proofs of these two stabilisation results, and then discuss the (mostly open) related question of calculating the stable homology in this more general setting. In particular, I will explain why the method of calculating the stable homology in the case of point particles does not generalise to higher-dimensional submanifolds, and show that two natural guesses for the stable homology are wrong. I will also describe some examples of natural finite-degree functors to which theorem (b) applies, related to the Burau and Lawrence-Krammer-Bigelow representations of the braid groups.

(Martin Palmer-Anghel) Universität Bonn
e-mail: `palmer@math.uni-bonn.de`