Homological stability for partitioned braid groups on surfaces

Talk at the GeMAT seminar, IMAR // Martin Palmer-Anghel // 24 August 2018

Abstract:

Braid groups were first introduced, implicitly, by Hurewicz, as the fundamental groups of configuration spaces of points in the plane (or, equivalently, the disc):

$$B_n = \pi_1(C_n(D^2)).$$

Using this viewpoint, one may generalise this to define the *surface braid groups* $B_n(S) = \pi_1(C_n(S))$ for any surface S. The topic of this talk will be the *partitioned braid groups* of a surface S, which are most easily defined using a slight modification of this viewpoint. Suppose that $n = tm$, and we have chosen a basepoint for $C_n(S)$ in which the n points are partitioned into t teams with m members each. The *partitioned braid group*

$$B_{m|t}(S) \leqslant B_n(S)$$

is the subgroup of all braids that respect this partition into teams (team members must end up together, although entire teams may swap positions). More formally, $B_{m|t}(S)$ is the preimage of a certain subgroup $(\Sigma_m)^t \rtimes \Sigma_t$ of Σ_n under the canonical projection $B_n(S) \to \Sigma_n$. It may also be interpreted as the fundamental group of a certain $\frac{n!}{t!(m!)^t}$-sheeted covering space of $C_n(S)$.

In the 1970s, V. I. Arnol'd, D. McDuff and G. Segal showed that, for any connected surface S with non-empty boundary, the surface braid groups $B_n(S)$ are *homologically stable*: their homology in any particular degree is independent of the number of strands n once n is large enough. The purpose of this talk is to present the proof of a generalisation of this result: for any fixed m and fixed homological degree i, the homology $H_i(B_{m|t}(S))$ is independent of the number of teams t once t is large enough.

As with many homological stability results, the main geometric input for the proof is the high-connectivity of certain simplicial complexes. I will first explain how one uses this input for the inductive proof of homological stability, and then sketch the proof of the high-connectivity of these simplicial complexes, using techniques discussed in a previous talk at GeMAT in July 2018 (but I will recall everything that we will need from that talk).

This talk represents joint work with TriThang Tran at the University of Melbourne.

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