Homological stability for partitioned braid groups on surfaces, II

Talk at the GeMAT seminar, IMAR // Martin Palmer-Anghel // 12 October 2018

Abstract:

This is a continuation of a talk that I gave at the GeMAT seminar in August 2018. The goal is to give a proof of the fact that, for any non-compact surface S, the so-called *partitioned braid groups* $B_{m|t}(S)$ are homologically stable as $t \to \infty$ (for any fixed $m \geqslant 1$), generalising a theorem of Arnol'd, McDuff and Segal from the 1970s that the classical braid groups on S (corresponding to the case $m = 1$) are homologically stable.

For the proof we use an axiomatisation by Hatcher and Wahl of an argument originally due to Quillen. This says, roughly, that if G_t is a sequence of groups acting on simplicial complexes X_t such that (a) the homotopy groups of X_t are zero in a range going to infinity as $t \to \infty$, and (b) certain additional geometric conditions on the action of G_t on X_t hold, then the sequence G_t is homologically stable.

In the previous part of this talk, last August, I took this argument as a black box, constructed the appropriate simplicial complexes X_t for the groups $G_t = B_{m|t}(S)$ and explained some of the ideas behind how one can prove condition (a), namely that these X_t have vanishing homotopy groups in a range of degrees diverging to infinity.

In this second part I will first review the definition of the partitioned braid groups $B_{m|t}(S)$ and the construction of the corresponding simplicial complexes X_t (so it will not be necessary to have been to my previous talk!). Then I will explain the inner workings of the black box in a little more detail, and in particular indicate where and why each of the assumptions is needed. Then, to complete the proof of homological stability for $B_{m|t}(S)$, I will explain how to prove the additional geometric conditions (b) needed for the argument.

This talk represents joint work with TriThang Tran at the University of Melbourne.

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