Calculating the stable homology of families of configuration spaces and other moduli spaces, I

Talk at the topology seminar, IMAR // Martin Palmer-Anghel // 22 March 2019

Abstract:

In this talk we will study the homology of configuration spaces on manifolds. When the underlying manifold M is connected and non-compact, there is a natural sequence of maps

$$\cdots \longrightarrow C_n(M) \longrightarrow C_{n+1}(M) \longrightarrow \cdots,$$

where $C_n(M)$ denotes the configuration space of n unordered points on M, and the homology of this sequence is known to be stable. It is therefore useful to identify the *stable homology*

$$\lim_{n\to\infty} H_*(C_n(M))$$

of this sequence with the homology of some other space that is in some sense “easier” to understand. In the first part of the talk, I will explain how this was done by D. McDuff in the 1970s.

In the second part, I will explain how to lift this result to *oriented configuration spaces* $C_n^+(M)$, which are the double coverings of $C_n(M)$ where each configuration is given an ordering modulo even permutations. Homological stability also holds in this case, and I will explain how to lift the result of McDuff to identify the stable homology

$$\lim_{n\to\infty} H_*(C_n^+(M))$$

in terms of a double covering of a section-space. This is done in two steps:

(1) When $M = \mathbb{R}^m$, using an analogue of the *group-completion theorem* for twisted homology.
(2) Deducing the general result from this special case, using an analogue of McDuff's *homology-fibration criterion* for twisted homology.

This part represents joint work with Jeremy Miller.

If time permits, there will also be a third part where I will discuss what happens when one replaces configurations of point-particles with configurations of other closed submanifolds – in this case, homological stability is known under certain conditions, but the stable homology remains much more mysterious.

The overall abstract for the series of talks is here.

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