Stability and stable homology for moduli spaces of disconnected submanifolds

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Configuration spaces.

Configuration spaces of unordered points in a manifold M are important objects in mathematics, with connections to knot theory (via the braid groups), homotopy theory and algebraic geometry. By results of D. McDuff and G. Segal [S1,McD,S2], their homology is known to *stabilise* as the number of points goes to infinity (assuming that M is connected and open), and the corresponding *stable homology* (i.e. the homology in the limit) is also known.

Moduli spaces of disconnected submanifolds.

One may consider a more general setting, in which configurations of point-particles are replaced by moduli spaces of disjoint unions of closed submanifolds of specified diffeomorphism type and isotopy class. The fundamental groups of these moduli spaces are *motion groups* (including as a special case the *loop-braid groups*), although they are typically not aspherical. In this talk I will discuss *homological stability* for these moduli spaces [K,P1], and outline a proof of this when the codimension is sufficiently large. In addition, I will explain some corollaries for diffeomorphism groups of manifolds with singularities [P2] and, if time permits, work in progress on identifying the *stable homology* in these settings.

References.

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