On homological stability for configuration-section spaces

Martin Palmer-Anghel // Talk at the Topology Seminar at Oxford on 27 April 2020

Abstract.

For a bundle over a manifold $E \to M$, the associated *configuration-section spaces* are spaces of configurations of points in M together with a section of E over the complement of the configuration. One often considers subspaces where the behaviour of the section near a configuration point – a kind of "monodromy" – is restricted or prescribed. These are examples of "non-local configuration spaces", and may be interpreted physically as moduli spaces of "fields with prescribed singularities" in an ambient manifold.

An important class of examples is given by *Hurwitz spaces*,[1] which are moduli spaces of branched G-coverings of the 2-disc, and which are homotopy equivalent to certain configuration-section spaces on the 2-disc. Ellenberg, Venkatesh and Westerland proved[2] that, under certain conditions, Hurwitz spaces are (rationally) homologically stable; from this they then deduced an asymptotic version of the Cohen-Lenstra conjecture for function fields, a purely number-theoretical result.

We will discuss another homological stability result for configuration-section spaces, which holds (with integral coefficients) whenever the base manifold M is connected and open. We will also show that the *stabilisation maps* are split-injective (in all degrees) whenever $\dim(M) \geqslant 3$ and M is either simply-connected or its *handle dimension* is at most $\dim(M) - 2$.

This represents joint work with Ulrike Tillmann.

[1] A. Hurwitz. *Über Riemann'sche Flächen mit gegebenen Verzweigungspunkten.* Math. Ann. 39, pp. 1–61. (1891)
[2] J. S. Ellenberg, A. Venkatesh and C. Westerland. *Homological stability for Hurwitz spaces and the Cohen-Lenstra conjecture over function fields.* Ann. of Math. (2) 183.3, pp. 729–786. (2016)