Homology of configuration-section spaces

Martin Palmer-Anghel // Topology Seminar, IMAR // 30 October 2020

Abstract.

For a manifold M equipped with a bundle E, the *configuration-section* spaces on (M, E) consist of configurations of points on M together with a section of E on the complement of the configuration. These may be thought of as moduli spaces of "fields" with singularities. One often considers subspaces where the behaviour of the field (section) is constrained in a neighbourhood of the singularities (particles), which may be thought of as restricting the allowed "charges" of the particles. As well as the evident physical interpretation, these spaces also include as examples the classical Hurwitz spaces, through which they have connections with number theory. In particular, Ellenberg, Venkatesh and Westerland [EVW] have recently proven an asymptotic version of the Cohen-Lenstra conjecture for function fields via a certain homological stability result for Hurwitz spaces.

I will talk about another homological stability result for configuration-section spaces, which is in a sense both more and less general than that of [EVW]. It is *more* general in the sense that it holds for any bundle over any connected, open manifold M (theirs is for certain trivial bundles over the 2-disc), but it is also *less* general in the sense that we assume a stronger condition on the allowed "charges" of the particles. Some examples of "fields" on the complement of a configuration that satisfy this condition – and hence to which our stability result applies – include:

- A non-vanishing vector field with prescribed winding number around each particle.
- A spin structure (if $\dim(M) \geqslant 4$), a string structure (if $\dim(M) \geqslant 6$), ...
- A flat connection for a principal G-bundle $P \to M$, if G is a Lie group and $\dim(M) = 3$.
- Certain fields corresponding to moduli spaces of "widely separated" magnetic monopoles.

Along the way I will also discuss explicit formulas (and pictures!) for the action of the braid group of a manifold M on the fibres of configuration-mapping spaces on M.

This represents joint work with Ulrike Tillmann and is based on the arxiv preprints 2007.11607 *and* 2007.11613.