Lower central series of braid-like groups

Martin Palmer-Anghel // Topology Seminar, IMAR // 22 and 29 January 2021

Abstract.

One of the most basic objects one needs to understand when studying the structure of a group G is its *lower central series*

$$G = \mathit{\Gamma}_1(G) \geqslant \mathit{\Gamma}_2(G) \geqslant \mathit{\Gamma}_3(G) \geqslant \cdots$$

which may be trivial, or it may contain deep information about its structure. How much information is contained in the lower central series $\mathit{\Gamma}_*(G)$ depends on whether – and if so when – this sequence *stops*, meaning that $\mathit{\Gamma}_i(G) = \mathit{\Gamma}_{i+1}(G)$ for some i, in which case we say that the sequence *stops at* $\mathit{\Gamma}_i$ (this automatically implies that $\mathit{\Gamma}_r(G) = \mathit{\Gamma}_{r+1}(G)$ for all $r \geqslant i$).

I will discuss a topological method for studying the stopping (or non-stopping) of lower central series of various "braid-like" groups, meaning classical, surface, virtual and welded braid groups, and generalisations of these. For example, I will explain why:

- the lower central series of the classical braid group $\mathbf{B}_n$ stops at $\mathit{\Gamma}_2$,
- the lower central series of the surface braid group $\mathbf{B}_2(S)$ does not stop, unless S is the disc, sphere or projective plane,
- the lower central series of the surface braid group $\mathbf{B}_n(S)$ stops at $\mathit{\Gamma}_3$ for $n \geqslant 3$,
- the lower central series of the welded braid group $\mathbf{wB}_n$ stops if and only if $n \notin \{2, 3\}$.

One particular motivation for studying these lower central series is the construction and study of *homological representations* of braid-like groups and mapping class groups, which naturally come in families depending on the lower central series of another associated braid-like group.

This represents joint work with Jacques Darné and Arthur Soulié.