Exercise sheet 1

Due before the lecture on Monday, 22 October 2018.

Exercise 1. (5 points) Let X be a pointed space. Show that ΣX is a co-H-group and that $\Sigma(\Sigma X)$ is a homotopy commutative co-H-group. Dualise your argument to show that ΩX is an H-group and that $\Omega(\Omega X)$ is a homotopy commutative H-group.

Exercise 2. (5 points) Weak equivalence is not a symmetric relation:

(a) Give an example of a weak equivalence $X \to Y$ for which there does not exist a weak equivalence $Y \to X$.

However, there is an equivalence relation $\simeq_w$ generated by weak equivalence: $X \simeq_w Y$ if there are spaces $X = X_1, X_2, \dots, X_n = Y$ with weak equivalences $X_i \to X_{i+1}$ or $X_i \leftarrow X_{i+1}$ for each i.

(b) Show that $X \simeq_w Y$ if and only if X and Y have a common CW approximation.

Exercise 3. (4 points) For $n \geq 0$, show that an n-connected, n-dimensional CW complex is contractible.

Exercise 4. (6 points)

(a) Show that the suspension of an acyclic CW complex (i.e. a CW complex whose reduced homology vanishes) is contractible.

(b) Show that a map between simply-connected CW complexes is a homotopy equivalence if its mapping cone is contractible.

(c) Provide an explicit counter-example to show that the connectivity assumption in (b) cannot be dropped.