

# Stability for moduli spaces of submanifolds and diffeomorphism groups

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## Abstract:

Configuration spaces  $C_n(M)$  of unordered points in an open, connected manifold  $M$  are known to be homologically stable: the sequence of groups  $H_i(C_n(M))$  is eventually constant for each fixed degree  $i$ . When  $M = \mathbb{R}^2$ , in which case the configuration spaces are classifying spaces for the braid groups, this was proven by V. I. Arnol'd, and in general it was proven by D. McDuff and G. Segal in the 1970s. A recently-proved consequence of this is homological stability for certain *symmetric diffeomorphism groups* of manifolds, with respect to the operation of connected sum (U. Tillmann).

I will talk about an extension of this result to *moduli spaces of disconnected submanifolds*  $C_{nP}(M)$ , a point in which consists of a “configuration” of  $n$  mutually isotopic embedded copies of a fixed “model” manifold  $P$  in  $M$ . This has two corollaries giving homological stability for certain diffeomorphism groups. One is for symmetric diffeomorphism groups with respect to *parametrised* connected sum (a generalisation of ordinary connected sum, including Dehn surgery in the case of 3-manifolds). Another is for diffeomorphism groups of manifolds with Baas-Sullivan singularities, with respect to the number of singularities of a given type.

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