Homological stability for moduli spaces of disconnected submanifolds and symmetric diffeomorphism groups

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Abstract:

An important property of the homology of the classical braid groups, due to V. I. Arnol'd, is that it *stabilises*, in the sense that $H_i(\beta_n)$ and $H_i(\beta_{n+1})$ are isomorphic if n is sufficiently large compared to i, where β_n denotes the braid group on n strands. The configuration space $C_n(\mathbb{R}^2)$ of n unordered points in the plane is a classifying space for β_n, so one may equivalently say that the homology of $C_n(\mathbb{R}^2)$ stabilises with respect to the number of points in a configuration. This was subsequently generalised by D. McDuff and G. Segal to an analogous result for the configuration spaces $C_n(M)$ on any open, connected manifold M.

A recently-discovered corollary, due to U. Tillmann, is that the homology of certain *symmetric diffeomorphism groups* of manifolds is also stable, with respect to the operation of connected sum.

After recalling these results, I will talk about an extension to "higher-dimensional configurations". First, homological stability for configuration spaces (of points) extends to *moduli spaces of disconnected submanifolds* $C_{nP}(M)$, a point in which consists of a "configuration" of n mutually isotopic embedded copies of a fixed "model" manifold P in M. This then allows us to generalise the above corollary to obtain stability for the homology of symmetric diffeomorphism groups with respect to *parametrised* (also called *parametric*) connected sum — this is a generalisation of the ordinary connected sum operation, including surgery (and Dehn surgery in the case of 3-manifolds) as special cases. In addition, there is a new corollary for the homology of diffeomorphism groups of manifolds with Baas-Sullivan singularities, with respect to the number of singularities of a given type.

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