Workshop "Cohomological study of mapping class groups and related topics"
IRMA, September 10-12, 2018

1 Schedule

Monday	Tuesday	Wednesday
09:30-10:00: Welcome coffee	09:30-10:30: **Krannich**	10:00-10:30: **Sato**
10:00-11:00: **Tsuji**	10:30-11:00: Coffee break	10:30-11:00: Coffee break
11:00-12:00: **Soulié**	11:00-12:00: **Nozaki**	11:00-12:00: **Petersen**
12:00-14:00: Lunch	12:00-14:00: Lunch	12:00-14:00: Lunch
14:00-15:00: **Ishibashi**	14:00-15:00: **Vera**	14:00-15:00: **Palmer**
15:00-15:30: Coffee break	15:00-15:30: Coffee break	15:00-15:30: Coffee break
15:30-16:30: **Djament**	15:30-16:30: **Darné**	15:30-16:30: **Sakasai**
	19:00: Dinner	

2 Titles and abstracts

- **Jacques Darné (Université de Lille)**
  The Andreadakis problem for some subgroups of $Aut(F_n)$
  The Andreadakis problem consists in comparing two filtrations on the group IA_n of automorphisms of the free group acting trivially on its abelianization (an algebraic analogue to the Torelli subgroup of the mapping class group). This difficult problem can be much easier when restricted to some subgroups of IA_n. Especially when these groups decompose nicely as iterated semi-direct products, as is the case for the group of triangular automorphisms studied by T. Satoh, or for the pure braid groups.

- **Aurélien Djament (CNRS - Université de Lille)**
  Stable homology of automorphism groups of free groups with twisted coefficients
  We will give an overview of recent results for the stable homology of automorphism groups of free groups with coefficients twisted by a suitable polynomial functor, gotten by using methods of functor homology. A first one (joint with C. Vespa, extending results first proved by independent topological methods) is a vanishing result for coefficients given by reduced polynomial covariant functors on the usual category gr of finitely generated free groups (a typical example of such a functor being a tensor power of the abelianization). A second one describes the stable homology for polynomial contravariant functors on gr - a typical example being the composition between Hom(-,Z) and a tensor power, where a complete computation is possible (the rational version of this result was proved also by independent topological methods by O. Randal-Williams). We will also discuss a conjecture for polynomial bivariant coefficients - typically, a tensor product between a covariant and a contravariant polynomial functor on gr.

- **Tsukasa Ishibashi (University of Tokyo)**
  Cluster realizations of Coxeter groups and higher Teichmuller theory
  Cluster algebras are commutative algebras associated with quivers, which describe the "positivity structures" appearing in many fields of mathematics. The automorphism group of the cluster algebra is called the cluster modular group: typical example is the mapping class group of an oriented marked surface. A realization of a group inside a cluster modular group allows one to understand that group by a unified combinatorial way.

  In this talk, we show that Coxeter groups can be realized in some cluster modular groups. Moreover, we show that our construction for finite type case is related to the Weyl group action on the moduli space of decorated local systems on a punctured disk. This talk is based on a joint work with Rei Inoue and Hironori Oya.

- **Manuel Krannich (University of Copenhagen)**
  Topological moduli spaces, E_2-algebras, and homological stability
  Since the seventies, many families of topological moduli spaces have been proven to stabilize homologically, including moduli spaces of Riemann surfaces (Harer), unordered configuration spaces (McDuff, Segal), and moduli spaces of higher-dimensional manifolds (Galatius, Randal-Williams). From the perspective of homotopy theory, a common structure these examples share is that of an E2-algebra, or at least of a module over such an algebra. In this talk, I will explain a framework which provides a uniform treatment of classical and new (twisted) homological stability results from this perspective. If time permits, I will also discuss how these results imply representation stability for related moduli spaces.

- **Yuta Nozaki (University of Tokyo)**
  An invariant of 3-manifolds via homology cobordisms
  For a closed 3-manifold X, we consider a topological invariant defined as the minimal integer g such that X is obtained as the closure of a homology cobordism over a surface of genus g. We prove that the invariant equals one for every lens space, which is contrast to the fact that some lens spaces do not admit any open book decomposition whose page is a surface of genus one. The proof is based on the Chebotarev density theorem and binary quadratic forms in number theory.

- **Martin Palmer (University of Bonn)**
  Homological stability for symmetric diffeomorphism groups and parametrised connected sum
  One very successful tool for studying the homology of diffeomorphism or mapping class groups of manifolds is *homological stability*: if the diffeomorphism (mapping class) groups of a sequence of manifolds are homologically stable, this reduces the calculation of their homology groups, in a range of degrees, to the calculation of the homology in the limit, which typically has more structure (for example that of a Hopf algebra), and is more amenable to explicit calculations.

  There are many results in the literature (including, among others, results of Harer, Ivanov, Boldsen, Hatcher-Wahl, Galatius-Randal-Williams, Perlmutter) on the homological stability of diffeomorphism or mapping class groups of sequences of manifolds of the form

  $$D^{p+q} \sharp (S^p \times S^q) \sharp (S^p \times S^q) \sharp \cdots$$

  obtained by iterating the operation of connected sum with a product of spheres, for various different values of (p, q). A recent result of Tillmann extends this to much more general sequences of manifolds of the form

  $$W \sharp N \sharp N \sharp \cdots$$

  at the expense of passing to certain subgroups of the full diffeomorphism groups of these manifolds, namely their *symmetric diffeomorphism groups*.

  I will present a generalisation of this result, where the operation $-\sharp-$ of connected sum is generalised to *parametrised connected sum* $-\underset{L}{\sharp}-$ *along a submanifold* L, an operation that includes surgery and Dehn surgery as special cases.

  A key input for the proof is homological stability for *moduli spaces of disconnected submanifolds*, a generalisation of configuration spaces whose points consist of configurations of isotopic copies of a given manifold L in the ambient manifold. I will give an overview of the main steps of the proof: how to reduce homological stability for symmetric diffeomorphism groups to homological stability for moduli spaces of disconnected submanifolds, and then the key ideas of the proof of the latter.

- **Dan Petersen (Stockholms Universitet)**
  Tautological rings with twisted coefficients
  Richard Hain has defined an "enlarged" tautological ring, which is a subalgebra of the cohomology ring of the genus g mapping class group with coefficients in the coordinate ring of the symplectic group. In joint work with Tavakol and Yin we calculated this ring completely in genus up to 4. Understanding this ring in fixed genus is equivalent to understanding all tautological rings of moduli spaces of pointed curves of this genus simultaneously. A conjecture of Yin regarding tautological classes on the universal jacobian variety is equivalent to the claim that this enlarged tautological

ring is isomorphic to the exterior algebra on the symplectic representation V_{111} modulo the ideal generated by the subrepresentation V_{22} occuring in the exterior square.

- **Takuya Sakasai (University of Tokyo)**
  Computations on Johnson homomorphisms
  There are two filtrations of the Torelli group: One is the lower central series and the other is the Johnson filtration. They are closely related to Johnson homomorphisms as well as finite type invariants of homology 3-spheres. We compare the associated graded Lie algebras of the filtrations and report our explicit computational results. Then we discuss some applications of our computations. This is a joint work with Shigeyuki Morita and Masaaki Suzuki.

- **Masatoshi Sato (Tokyo Denki University)**
  On the rational homology group of the genus three handlebody mapping class group
  The handlebody mapping class group acts on a simplicial complex called the meridian disk complex, and McCullough showed that it is contractible. When the genus is three, the stabilizer of a vertex is the mapping class group of a genus two handlebody with two marked disks, and the stabilizer of one of edges is the mapping class group of a solid torus with four marked disks. In this talk, we explain a method to compute low-dimensional rational homology groups of these stabilizers and the genus three handlebody mapping class group.

- **Arthur Soulié (Université de Strasbourg)**
  Stable homology and mapping class groups.
  In 2017, Randal-Williams and Wahl set a general framework to prove homological stability with twisted coefficients for different families of groups, including automorphism groups of free groups, mapping class groups of orientable and non-orientable surfaces or mapping class groups of 3-manifolds. The twisted coefficients they use satisfy a "polynomial" condition which I will recall. Moreover, when homological stability is satisfied, computing the colimit of the homology groups gives the stable value. I will present a general method to tackle this problem. This includes presenting a general result of splitting of the stable homology with twisted coefficients thanks to functor homology I extended from the framework of Djament and Vespa for the situation of mapping class group of surfaces. Then I will give some explicit results for some particular coefficients.

- **Shunsuke Tsuji (Kyoto University / JSPS)**
  Formulas for the actions of Dehn twists on skein algebras and their applications
  We give two explicit formulas for the actions of the Dehn twists along simple closed curves of a surface on the completed Kauffman bracket skein modules and the completed HOMFLY-PT type skein modules of the surface. As an application, using these formulas, we construct some filtrations of the Torelli group. Furthermore, we construct some invariants for an integral homology 3-sphere.

- **Anderson Vera (Université de Strasbourg)**
  Johnson type homomorphisms and the LMO functor
  One of the main objects associated to a surface S is the mapping class group MCG(S). This group plays an important role in the study of 3-manifolds through Heegaard splittings. Reciprocally, the topological invariants of 3-manifolds can be used to obtain interesting representations of MCG(S). One possible approach to the study of MCG(S) is to consider its action on the fundamental group P of the surface or on some subgroups of P. This way, we can obtain some kind of filtrations of MCG(S) and homomorphisms, called Johnson type homomorphisms, which take values in certain spaces of diagrams. These spaces of diagrams are quotients of the target space of an universal invariant of 3-manifolds: The Le-Murakami-Ohtsuki invariant. Hence it is natural to ask what is the relation between the Johnson type homomorphisms and the LMO invariant. The answer is well known in the case of the Torelli group and the usual Johnson homomorphisms. In this talk we consider two other different filtrations of MCG(S) introduced by Levine and Habiro-Massuyeau, respectively. We show that the respective Johnson homomorphisms can also be deduced from the LMO invariant.

3 Dinner

A dinner is organized on Tuesday, September 11th at 7PM at the restaurant Le Baeckeoffe d'Alsace, 14 rue des moulins.