Calculating the stable homology of families of configuration spaces and other moduli spaces, II

Talk at the GeMAT seminar, IMAR // Martin Palmer-Anghel // 26 April 2019

Abstract:

The first talk in this series (link) was about identifying the homology

$$\lim_{n\to\infty} H_*(C_n(M))$$

of configuration spaces on a manifold M (assumed to be smooth, connected and *non-compact*), in the limit as the number of points n goes to infinity. Since this sequence of spaces is known to be homologically stable, this is usually called the *stable homology* of configuration spaces on M. We saw McDuff's proof of this calculation, and also considered how to lift it to oriented configuration spaces, using a form of the *group-completion theorem.*

In part 1 of this talk, we will recap this briefly, and then explain a more general version of the group-completion theorem and some of the ideas of its proof. In part 2 we will then use this to give an alternative proof of McDuff's result, following a strategy of proof due to Galatius and Randal-Williams.[1]

In part 3 I will start to explain how Galatius and Randal-Williams use this strategy to prove the *Madsen-Weiss theorem*, identifying the stable (co)homology of the moduli spaces of compact, connected, orientable surfaces (a.k.a. the moduli spaces of complex curves), which in particular verifies the *Mumford conjecture* that their stable rational cohomology is a free polynomial algebra on certain geometrically-defined classes. For this part of the talk I will mainly follow the paper *Monoids of moduli spaces of manifolds* of Galatius-Randal-Williams.[2] (The proof is quite long and involved, so it will also occupy most of talk III of this series.)

The overall abstract for the series of talks is here.[3]

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[1] A small technical note: In the previous talk, we saw how to prove that the scanning map for configuration spaces on manifolds is acyclic (a twisted homology equivalence), using the group-completion theorem plus McDuff's theorem that the scanning map is a homology equivalence (with untwisted coefficients). This time, we will prove that the scanning map is acyclic just using the group-completion theorem, so this in particular re-proves McDuff's theorem.

[2] This is one of at least 4 published proofs of the Madsen-Weiss theorem. A significant feature of the proof of Galatius-Randal-Williams is that it is completely independent of knowing homological stability for configuration spaces, unlike the original proof of Madsen-Weiss and the proof of Galatius-Madsen-Tillmann-Weiss.

[3] URL: `mdp.ac/files/1900-talk-abstract-IMAR-series-on-stable-homology.pdf`