

# On homological stability for configuration-section spaces

Martin Palmer-Anghel // Talk at the Algebraic Topology Seminar of Paris 13 on 12 March 2020

## Abstract.

*Configuration-mapping spaces*, introduced by Ellenberg, Venkatesh and Westerland,<sup>1</sup> are spaces of configurations of points on a manifold, together with a continuous map from the complement of the configuration to a fixed space, with prescribed “monodromy” in a neighbourhood of the configuration points. This naturally generalises to *configuration-section spaces*, where the complement is equipped with a section of a given bundle over the manifold, such as a non-vanishing vector field. These spaces may be interpreted physically as spaces of “fields” in an ambient manifold, which are permitted to be singular at a finite number of points, with prescribed behaviour near the singularities.

An important class of examples is given by *Hurwitz spaces*,<sup>2</sup> which are moduli spaces of branched  $G$ -coverings of the 2-disc, and which are homotopy equivalent to certain configuration-mapping spaces on the 2-disc. Ellenberg, Venkatesh and Westerland proved<sup>3</sup> that, under certain conditions, Hurwitz spaces are (rationally) homologically stable; from this they then deduced an asymptotic version of the Cohen-Lenstra conjecture for function fields, a purely number-theoretical result.

We will discuss a higher-dimensional analogue of their stability result, namely: homological stability (with integral coefficients) for configuration-section spaces on an ambient manifold  $M$  of dimension at least 3, as long as  $M$  is either simply-connected or its geometric dimension and its handle dimension differ by at least 2.

*This represents joint work with Ulrike Tillmann.*

---

<sup>1</sup> J. S. Ellenberg, A. Venkatesh and C. Westerland. *Homological stability for Hurwitz spaces and the Cohen-Lenstra conjecture over function fields, II*. ArXiv:1212.0923v1. (2012)

<sup>2</sup> A. Hurwitz. *Über Riemann'sche Flächen mit gegebenen Verzweigungspunkten*. Math. Ann. 39, pp. 1–61. (1891)

<sup>3</sup> J. S. Ellenberg, A. Venkatesh and C. Westerland. *Homological stability for Hurwitz spaces and the Cohen-Lenstra conjecture over function fields*. Ann. of Math. (2) 183.3, pp. 729–786. (2016)