

Spectrification of Khovanov homology

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Abstract.

The *Jones polynomial*, discovered by V. Jones in 1984, is a function

$$J: \{\text{isotopy classes of links}\} \longrightarrow \mathbb{Z}[t^{\pm 1}]$$

sending isotopy classes of links to Laurent polynomials in one variable. This was *categorified* by M. Khovanov in 2000 to a function (*Khovanov homology*)

$$Kh: \{\text{isotopy classes of links}\} \longrightarrow \{\text{bigraded abelian groups}\}$$

such that $J = E_* \circ Kh$ where E_* is the (graded) Euler characteristic. This was further *spectrified* by R. Lipshitz and S. Sarkar in 2011 to a function (*Khovanov spectrum*)

$$\mathcal{X}: \{\text{isotopy classes of links}\} \longrightarrow \{\text{graded spectra}\}$$

such that $Kh = H_{**} \circ \mathcal{X}$, where H_{**} denotes ordinary homology of (graded) spectra.

In this series of expository talks, I will aim to describe:

- Lipshitz and Sarkar's original construction of the Khovanov spectrum $\mathcal{X}(L)$ of a link L , using the *framed flow categories* of Cohen-Jones-Segal, and also why $\mathcal{X}(L)$ contains (strictly) more information than $Kh(L)$.
- A second, simpler and more combinatorial construction of $\mathcal{X}(L)$, due to Lawson-Lipshitz-Sarkar, using the *Burnside category* and building on earlier work of Hu-Kriz-Kriz.
- An extension of \mathcal{X} to *tangles* (spectrifying Khovanov's extension of Kh to tangles).
- An extension of \mathcal{X} to a *functor* (up to a sign) on the category of tangles and tangle cobordisms (spectrifying the Kh functor-up-to-a-sign due to Jacobsson, Khovanov and Bar-Natan).

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