Homological mapping class group representations and lower central series

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Abstract.

A long-standing open question about mapping class groups of surfaces is whether they are linear, i.e. act faithfully on finite-dimensional vector spaces. In genus zero, for the braid groups, the answer is yes, as proven by Bigelow and Krammer using one of the family of Lawrence representations of the braid groups. Motivated by this, I will describe an analogue of the family of Lawrence representations for higher-genus surfaces – depending on a chosen representation V of the discrete Heisenberg group. A subtlety is that these mapping class group representations are in general twisted, essentially as a consequence of the non-commutativity of the discrete Heisenberg group. However, I will explain how to untwist them for particular choices of V (and for any V if we restrict to the Torelli group). This all represents joint work with Christian Blanchet and Awais Shaukat. The appearance of the discrete Heisenberg group in the construction arises from the study of the lower central series of (partitioned) surface braid groups: I will also outline recent joint work with Jacques Darné and Arthur Soulié that answers the stopping question for these lower central series.