Untwisting Heisenberg homological representations of mapping class groups, II

Martin Palmer-Anghel // Topology Seminar, IMAR // 5 September 2023

Abstract.

A long-standing open question about mapping class groups of surfaces is whether they are *linear*, i.e. act faithfully on finite-dimensional vector spaces. In genus zero, for the braid groups, the answer is yes, as proven by Bigelow and Krammer using one of the family of Lawrence representations of the braid groups.

In contrast, the question is still wide open for higher genus surfaces. Motivated by this, in recent joint work with Christian Blanchet and Awais Shaukat, we have constructed analogues of the family of Lawrence representations for mapping class groups of higher-genus surfaces – depending on a chosen representation V of the discrete Heisenberg group $\mathcal{H}_g$.

However, these mapping class group representations are in general *twisted*, in a sense that I will explain precisely. There are four settings where we are able to untwist and obtain genuine linear representations:

1. If we restrict to the Torelli group (for any choice of V).
2. When $V = \mathcal{H}_g \oplus \mathbb{Z}$ is the linearisation of the affine translation action of $\mathcal{H}_g$ on itself.
3. When V is the *Schrödinger representation* of $\mathcal{H}_g$ and either:
   (a) we pass to a central extension of the mapping class group;
   (b) we restrict to an *Earle-Morita subgroup* of the mapping class group.

In part I (in August), I explained the first point above: untwisting on the Torelli group. In this talk (part II), I will first recall the general construction of the twisted representations of the mapping class groups – making this talk independent of part I – and then explain the second and third points: untwisting results for special choices of the representation V of the Heisenberg group.

This all represents joint work with Christian Blanchet and Awais Shaukat and is based on the two preprints arXiv:2109.00515 and arXiv:2306.08614.