

Moments of families of quadratic L-functions via scanning (after Bergström, Diaconu, Petersen and Westerland)

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Abstract.

In 2005, Conrey–Farmer–Keating–Rubinstein–Snaith [CFKRS] gave a general heuristic for conjecturing the integral moments of families of L-functions and applied it to give explicit conjectures for – among others – the Riemann zeta-function and the family of primitive Dirichlet L-functions. Their heuristic was later adapted by Andrade–Keating [AK] to the function field setting to give conjectural asymptotic formulas for the integral moments of the family of quadratic, primitive Dirichlet L-functions over function fields.

In a recent preprint, Bergström–Diaconu–Petersen–Westerland [BDPW] proved this conjecture of [AK], modulo another – purely topological – conjecture. The latter (topological) conjecture is the assertion that the homology of the braid groups, with specific families of twisted coefficients, stabilises uniformly; a proof of it has very recently been announced by Miller–Patz–Petersen–Randal-Williams, using work-in-progress by Sierra–Wahl.

I will describe the strategy of proof of [BDPW], focusing in particular on their use of *scanning* techniques, which go back to work of Segal [S] in the 1970s.

References.

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