Outline
Configuration spaces
(Points)
(Discs)
Braid groups and low-dim. topology
(Alexander/Markov)
Hurwitz spaces and number theory
Magnetic monopole moduli spaces
(Donaldson)
(Kottke-Singer)
Summary

Configuration spaces and their applications

Martin Palmer-Anghel

(Mathematical Institute of the Romanian Academy (IMAR), Bucharest)

18 January 2024

Outline of the talk

- Configuration spaces — What are they? — Examples
- Connections to low-dimensional topology via braid groups
- Connections to number theory via Hurwitz spaces
- Connections to physics via magnetic monopole moduli spaces

Configuration spaces

Configuration spaces

What are they?

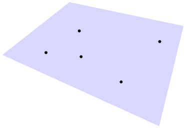
Spaces each of whose points encapsulates the complete state of a given system (i.e. its *parameters* or *degrees of freedom*), for example:

Configuration spaces

What are they?

Spaces each of whose points encapsulates the complete state of a given system (i.e. its *parameters* or *degrees of freedom*), for example:

- positions of particles in an ambient space

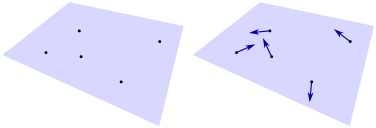


Configuration spaces

What are they?

Spaces each of whose points encapsulates the complete state of a given system (i.e. its *parameters* or *degrees of freedom*), for example:

- positions of particles in an ambient space
- positions *and momenta* of particles in an ambient space (classical mechanics)

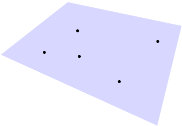


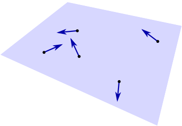
Configuration spaces

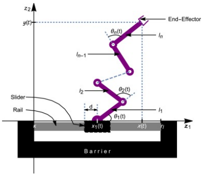
What are they?

Spaces each of whose points encapsulates the complete state of a given system (i.e. its *parameters* or *degrees of freedom*), for example:

- positions of particles in an ambient space
- positions *and momenta* of particles in an ambient space (classical mechanics)
- arrangements of a mechanical linkage (e.g. robot arm)





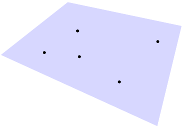


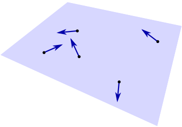
Configuration spaces

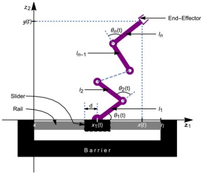
What are they?

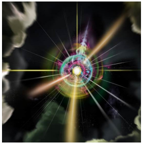
Spaces each of whose points encapsulates the complete state of a given system (i.e. its *parameters* or *degrees of freedom*), for example:

- positions of particles in an ambient space
- positions *and momenta* of particles in an ambient space (classical mechanics)
- arrangements of a mechanical linkage (e.g. robot arm)
- magnetic monopoles [we will see this later]









Outline
Configuration spaces
(Points)
(Discs)
Braid groups and low-dim. topology
(Alexander/Markov)
Hurwitz spaces and number theory
Magnetic monopole moduli spaces
(Donaldson)
(Kottke-Singer)
Summary

Configuration spaces

Points

Outline
Configuration spaces
(Points)
(Discs)
Braid groups and low-dim. topology
(Alexander/Markov)
Hurwitz spaces and number theory
Magnetic monopole moduli spaces
(Donaldson)
(Kottke-Singer)
Summary

Configuration spaces

Points

Fix integers $n \geqslant 1$ and $d \geqslant 1$.
The collection of all possible "configurations" of n distinct points in d-dimensional Euclidean space forms the *configuration space* $C_n(\mathbf{R}^d)$.

(We can also consider more general "ambient" spaces than Euclidean spaces...)

Outline
Configuration spaces
(Points)
(Discs)
Braid groups and low-dim. topology
(Alexander/Markov)
Hurwitz spaces and number theory
Magnetic monopole moduli spaces
(Donaldson)
(Kottke-Singer)
Summary

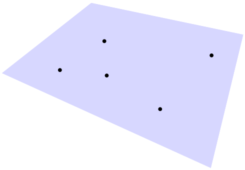
Configuration spaces

Points

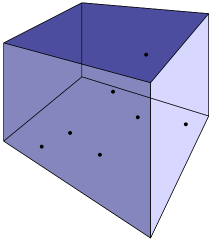
Fix integers $n \geqslant 1$ and $d \geqslant 1$.
The collection of all possible "configurations" of n distinct points in d-dimensional Euclidean space forms the *configuration space* $C_n(\mathbf{R}^d)$.

(We can also consider more general "ambient" spaces than Euclidean spaces...)

For example:



$\in C_5(\mathbf{R}^2)$



$\in C_7(\mathbf{R}^3)$

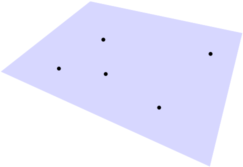
Configuration spaces

Points

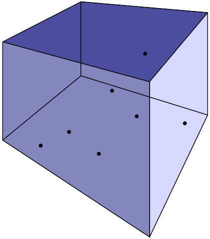
Fix integers $n \geqslant 1$ and $d \geqslant 1$.
The collection of all possible "configurations" of n distinct points in d-dimensional Euclidean space forms the *configuration space* $C_n(\mathbf{R}^d)$.

(We can also consider more general "ambient" spaces than Euclidean spaces...)

For example:



$\in C_5(\mathbf{R}^2)$



$\in C_7(\mathbf{R}^3)$

$(n = 2)$: configuration $\in C_2(\mathbf{R}^2) \longleftrightarrow$ (centre $\in \mathbf{R}^2$, separation $\in \mathbf{R}_+$, angle $\in \mathbf{S}^1$)

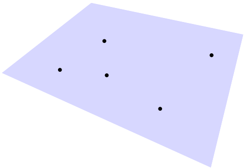
Configuration spaces

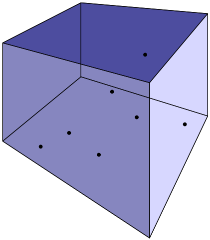
Points

Fix integers $n \geqslant 1$ and $d \geqslant 1$.
The collection of all possible "configurations" of n distinct points in d-dimensional Euclidean space forms the *configuration space $C_n(\mathbf{R}^d)$*.

(We can also consider more general "ambient" spaces than Euclidean spaces...)

For example:


$\in C_5(\mathbf{R}^2)$


$\in C_7(\mathbf{R}^3)$

$(n = 2)$: configuration $\in C_2(\mathbf{R}^2) \longleftrightarrow$ (centre $\in \mathbf{R}^2$, separation $\in \mathbf{R}_+$, angle $\in \mathbf{S}^1$)
$C_2(\mathbf{R}^2) \cong$ "thickened circle"

Configuration spaces and their applications

Outline

Configuration spaces

(Points)

(Discs)

Braid groups and low-dim. topology

(Alexander/Markov)

Hurwitz spaces and number theory

Magnetic monopole moduli spaces

(Donaldson)

(Kottke-Singer)

Summary

Discs in a square

Configuration spaces and their applications

Outline

Configuration spaces

(Points)

(Discs)

Braid groups and low-dim. topology

(Alexander/Markov)

Hurwitz spaces and number theory

Magnetic monopole moduli spaces

(Donaldson)

(Kottke-Singer)

Summary

Discs in a square

Fix an integer $n \geqslant 1$ and a positive number $\epsilon > 0$.
$C_n^{\epsilon}(\mathbf{Sq})$ = all configurations of n non-overlapping discs of radius ϵ inside the unit square.

Outline
Configuration spaces
(Points)
(Discs)
Braid groups and low-dim. topology
(Alexander/Markov)
Hurwitz spaces and number theory
Magnetic monopole moduli spaces
(Donaldson)
(Kottke-Singer)
Summary

Configuration spaces

Discs in a square

Fix an integer $n \geqslant 1$ and a positive number $\epsilon > 0$.
$C_n^\epsilon(\mathbf{Sq})$ = all configurations of n non-overlapping discs of radius ϵ inside the unit square.

The "shape" of $C_n^\epsilon(\mathbf{Sq})$ has many "phase transitions" as $\epsilon \to 0$.

Configuration spaces

Discs in a square

Fix an integer $n \geqslant 1$ and a positive number $\epsilon > 0$.
$C_n^\epsilon(\mathbf{Sq})$ = all configurations of n non-overlapping discs of radius ϵ inside the unit square.

The "shape" of $C_n^\epsilon(\mathbf{Sq})$ has many "phase transitions" as $\epsilon \to 0$.

For $n = 3$ there are 4 phase transitions with critical configurations:

For $n = 4$ there are 8 phase transitions with critical configurations:

(Exercise: work out the critical values of ϵ from these pictures!)

Configuration spaces

Discs in a square

Fix an integer $n \geqslant 1$ and a positive number $\epsilon > 0$.
$C_n^\epsilon(\mathbf{Sq})$ = all configurations of n non-overlapping discs of radius ϵ inside the unit square.

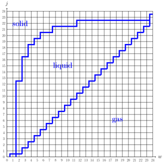
The "shape" of $C_n^\epsilon(\mathbf{Sq})$ has many "phase transitions" as $\epsilon \to 0$.

For $n = 3$ there are 4 phase transitions with critical configurations:

For $n = 4$ there are 8 phase transitions with critical configurations:

(Exercise: work out the critical values of ϵ from these pictures!)

Counting the number of "d-dimensional holes" in $C_n^\epsilon(\mathbf{Sq})$ and plotting a graph of d against ϵ (fixing n) leads to *phase diagrams* such as the one on the right.



Braid groups and low-dimensional topology

Braid groups and low-dimensional topology

Braid groups

Let's go back to configurations of points in the plane: $C_n(\mathbf{R}^2)$.

Braid groups and low-dimensional topology

Braid groups

Let's go back to configurations of points in the plane: $C_n(\mathbf{R}^2)$.

Fix a configuration c_0 and look at *loops of configurations* starting and ending at c_0.
$\rightsquigarrow$ the *braid group* $\mathbf{B}_n = \pi_1(C_n(\mathbf{R}^2))$ (see last week's talk by P. Bellingeri)

Braid groups and low-dimensional topology

Braid groups

Let's go back to configurations of points in the plane: $C_n(\mathbf{R}^2)$.

Fix a configuration c_0 and look at *loops of configurations* starting and ending at c_0.
$\rightsquigarrow$ the *braid group* $\mathbf{B}_n = \pi_1(C_n(\mathbf{R}^2))$ (see last week's talk by P. Bellingeri)

Each loop in $\mathbf{B}_2$ is fully described by an integer (*winding number*):

 $\longleftrightarrow \quad 1 \in \mathbf{Z}$

Outline
Configuration spaces
(Points)
(Discs)
Braid groups and low-dim. topology
(Alexander/Markov)
Hurwitz spaces and number theory
Magnetic monopole moduli spaces
(Donaldson)
(Kottke-Singer)
Summary

Braid groups and low-dimensional topology

Braid groups

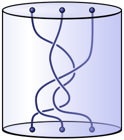
Let's go back to configurations of points in the plane: $C_n(\mathbf{R}^2)$.

Fix a configuration c_0 and look at *loops of configurations* starting and ending at c_0.
$\rightsquigarrow$ the *braid group* $\mathbf{B}_n = \pi_1(C_n(\mathbf{R}^2))$ (see last week's talk by P. Bellingeri)

Each loop in $\mathbf{B}_2$ is fully described by an integer (*winding number*):

 $\longleftrightarrow \quad 1 \in \mathbf{Z}$

For $n \geqslant 3$, loops in $\mathbf{B}_n$ become much more complicated, for example:



Outline
Configuration spaces
(Points)
(Discs)
Braid groups and low-dim. topology
(Alexander/Markov)
Hurwitz spaces and number theory
Magnetic monopole moduli spaces
(Donaldson)
(Kottke-Singer)
Summary

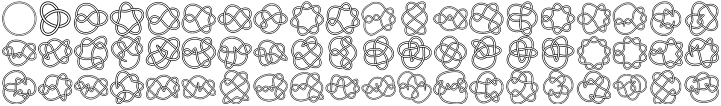
Braid groups and low-dimensional topology

Connections to low-dimensional topology

Braid groups and low-dimensional topology

Connections to low-dimensional topology

A lot of low-dimensional topology is concerned with *knots* and *links* — collections of closed loops in Euclidean space $\mathbf{R}^3$. For example:



Configuration spaces and their applications

Outline

Configuration spaces
(Points)
(Discs)

Braid groups and low-dim. topology
(Alexander/Markov)

Hurwitz spaces and number theory

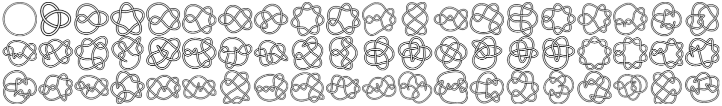
Magnetic monopole moduli spaces
(Donaldson)
(Kottke-Singer)

Summary

Braid groups and low-dimensional topology

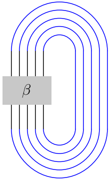
Connections to low-dimensional topology

A lot of low-dimensional topology is concerned with *knots* and *links* — collections of closed loops in Euclidean space $\mathbf{R}^3$. For example:



Any braid can be *closed* to form a link:

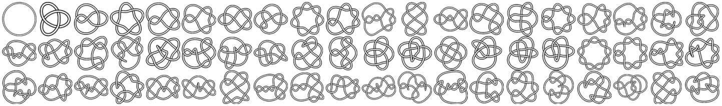
$$\bigcup_{n\geqslant 1} \mathbf{B}_n \xrightarrow{\ cl\ } \{\text{links}\}$$



Braid groups and low-dimensional topology

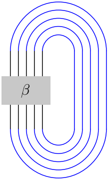
Connections to low-dimensional topology

A lot of low-dimensional topology is concerned with *knots* and *links* — collections of closed loops in Euclidean space $\mathbf{R}^3$. For example:



Any braid can be *closed* to form a link:

$$\bigcup_{n\geqslant 1} \mathbf{B}_n \xrightarrow{\;cl\;} \{\text{links}\}$$



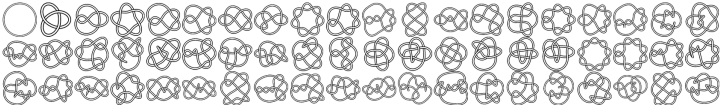
Theorem (Alexander):

- Every link is $cl(\beta)$ for a braid β.

Braid groups and low-dimensional topology

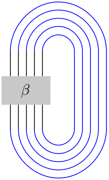
Connections to low-dimensional topology

A lot of low-dimensional topology is concerned with *knots* and *links* — collections of closed loops in Euclidean space $\mathbf{R}^3$. For example:



Any braid can be *closed* to form a link:

$$\bigcup_{n\geqslant 1} \mathbf{B}_n \xrightarrow{\;cl\;} \{\text{links}\}$$



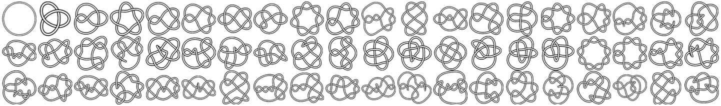
Theorem (Alexander and Markov):

- Every link is $cl(\beta)$ for a braid β.
- If $cl(\beta_1) = cl(\beta_2)$ then $\beta_1 \rightsquigarrow \beta_2$ via the *Markov moves*.

Braid groups and low-dimensional topology

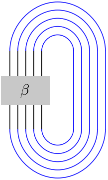
Connections to low-dimensional topology

A lot of low-dimensional topology is concerned with *knots* and *links* — collections of closed loops in Euclidean space $\mathbf{R}^3$. For example:



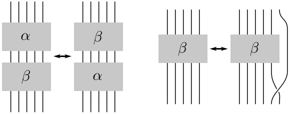
Any braid can be *closed* to form a link:

$$\bigcup_{n \geqslant 1} \mathbf{B}_n \xrightarrow{\;cl\;} \{\text{links}\}$$



Theorem (Alexander and Markov):

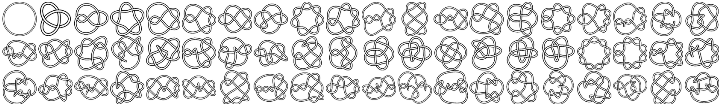
- Every link is $cl(\beta)$ for a braid β.
- If $cl(\beta_1) = cl(\beta_2)$ then $\beta_1 \rightsquigarrow \beta_2$ via the *Markov moves*.



Braid groups and low-dimensional topology

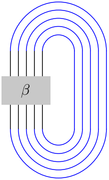
Connections to low-dimensional topology

A lot of low-dimensional topology is concerned with *knots* and *links* — collections of closed loops in Euclidean space $\mathbf{R}^3$. For example:



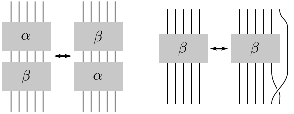
Any braid can be *closed* to form a link:

$$\bigcup_{n\geqslant 1} \mathbf{B}_n \xrightarrow{\;cl\;} \{\text{links}\}$$



Theorem (Alexander and Markov):

- Every link is $cl(\beta)$ for a braid β.
- If $cl(\beta_1) = cl(\beta_2)$ then $\beta_1 \rightsquigarrow \beta_2$ via the *Markov moves*.



Understanding braids and Markov moves $\rightsquigarrow$ understanding links.

Hurwitz spaces and number theory

Hurwitz spaces and number theory

We can enrich $C_n(\mathbf{R}^2)$ by adding some extra data:

Outline
Configuration spaces
(Points)
(Discs)
Braid groups and low-dim. topology
(Alexander/Markov)
Hurwitz spaces and number theory
Magnetic monopole moduli spaces
(Donaldson)
(Kottke-Singer)
Summary

Hurwitz spaces and number theory

We can enrich $C_n(\mathbf{R}^2)$ by adding some extra data:

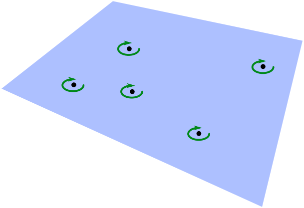
- a *field* defined on the complement of the configuration
- with *specified charge* (local behaviour) near each of the n points

Hurwitz spaces and number theory

We can enrich $C_n(\mathbf{R}^2)$ by adding some extra data:

- a *field* defined on the complement of the configuration
- with *specified charge* (local behaviour) near each of the n points

These are **Hurwitz spaces**: $\mathrm{Hur}^{c}_{G,n}$

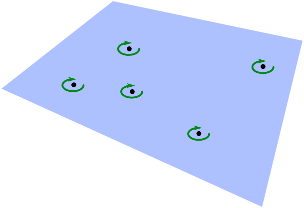


Hurwitz spaces and number theory

We can enrich $C_n(\mathbf{R}^2)$ by adding some extra data:

- a *field* defined on the complement of the configuration
- with *specified charge* (local behaviour) near each of the n points

These are **Hurwitz spaces**: $\mathrm{Hur}^c_{G,n}$



Theorem (Ellenberg, Venkatesh and Westerland 2016):
Solve a deep question in analytic number theory by understanding *how the shape of the space* $\mathrm{Hur}^c_{G,n}$ *changes as* $n \to \infty$.

(Here, *shape* means the *number of d-dimensional holes* for each $d \geqslant 0$.)

Magnetic monopole moduli spaces

Magnetic monopoles

Outline
Configuration spaces
(Points)
(Discs)
Braid groups and low-dim. topology
(Alexander/Markov)
Hurwitz spaces and number theory
Magnetic monopole moduli spaces
(Donaldson)
(Kottke-Singer)
Summary

Magnetic monopole moduli spaces

Magnetic monopoles

The Maxwell equations of electromagnetism are asymmetric.
There is a symmetrised version — and if we set the *magnetic charge density* to zero, we get back the classical (asymmetric) version.

Magnetic monopole moduli spaces

Magnetic monopoles

The Maxwell equations of electromagnetism are asymmetric.
There is a symmetrised version — and if we set the *magnetic charge density* to zero, we get back the classical (asymmetric) version.
Dirac found (singular) solutions with non-zero magnetic charges — these are *magnetic monopoles*.
(Singular = undefined on some 1-dimensional subsets of $\mathbf{R}^3$)

Magnetic monopole moduli spaces

Magnetic monopoles

The Maxwell equations of electromagnetism are asymmetric.
There is a symmetrised version — and if we set the *magnetic charge density* to zero, we get back the classical (asymmetric) version.
Dirac found (singular) solutions with non-zero magnetic charges — these are *magnetic monopoles*.
(Singular = undefined on some 1-dimensional subsets of $\mathbf{R}^3$)

Bogomolny equations: a different model for magnetic monopoles.
Non-singular solutions (defined on all of $\mathbf{R}^3$) behave at large distances like Dirac's singular solutions of the symmetrised Maxwell equations.

Outline
Configuration spaces
(Points)
(Discs)
Braid groups and low-dim. topology
(Alexander/Markov)
Hurwitz spaces and number theory
Magnetic monopole moduli spaces
(Donaldson)
(Kottke-Singer)
Summary

Magnetic monopole moduli spaces

Magnetic monopoles

The Maxwell equations of electromagnetism are asymmetric.
There is a symmetrised version — and if we set the *magnetic charge density* to zero, we get back the classical (asymmetric) version.
Dirac found (singular) solutions with non-zero magnetic charges — these are *magnetic monopoles*.
(Singular = undefined on some 1-dimensional subsets of $\mathbf{R}^3$)

Bogomolny equations: a different model for magnetic monopoles.
Non-singular solutions (defined on all of $\mathbf{R}^3$) behave at large distances like Dirac's singular solutions of the symmetrised Maxwell equations.

Magnetic monopole moduli spaces

For a positive integer n,
$\mathcal{M}_n =$ all solutions to the Bogomolny equations of total charge n

Magnetic monopole moduli spaces

Magnetic monopoles

The Maxwell equations of electromagnetism are asymmetric.
There is a symmetrised version — and if we set the *magnetic charge density* to zero, we get back the classical (asymmetric) version.
Dirac found (singular) solutions with non-zero magnetic charges — these are *magnetic monopoles*.
(Singular = undefined on some 1-dimensional subsets of $\mathbf{R}^3$)

Bogomolny equations: a different model for magnetic monopoles.
Non-singular solutions (defined on all of $\mathbf{R}^3$) behave at large distances like Dirac's singular solutions of the symmetrised Maxwell equations.

Magnetic monopole moduli spaces

For a positive integer n,
$\mathcal{M}_n =$ all solutions to the Bogomolny equations of total charge n

$\mathcal{M}_1 \cong \mathbf{R}^3 \times \mathbf{S}^1$ ("thickened circle")
$\mathcal{M}_n$ is $4n$-dimensional

Outline
Configuration spaces
(Points)
(Discs)
Braid groups and low-dim. topology
(Alexander/Markov)
Hurwitz spaces and number theory
Magnetic monopole moduli spaces
(Donaldson)
(Kottke-Singer)
Summary

Magnetic monopole moduli spaces

Outline
Configuration spaces
(Points)
(Discs)
Braid groups and low-dim. topology
(Alexander/Markov)
Hurwitz spaces and number theory
Magnetic monopole moduli spaces
(Donaldson)
(Kottke-Singer)
Summary

Magnetic monopole moduli spaces

Theorem (Donaldson 1984):
The magnetic monopole moduli space $\mathcal{M}_n$ is a configuration space.

Magnetic monopole moduli spaces

Theorem (Donaldson 1984):
The magnetic monopole moduli space $\mathcal{M}_n$ is a configuration space.
Precisely, it is the configuration space of:

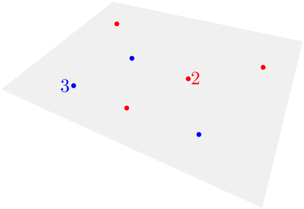
- n red points in $\mathbf{R}^2$ *that are allowed to collide*,
- n blue points in $\mathbf{R}^2$ *that are allowed to collide*,
- but red points are not allowed to collide with blue points.

Magnetic monopole moduli spaces

Theorem (Donaldson 1984):
The magnetic monopole moduli space $\mathcal{M}_n$ is a configuration space.
Precisely, it is the configuration space of:

- n red points in $\mathbf{R}^2$ *that are allowed to collide*,
- n blue points in $\mathbf{R}^2$ *that are allowed to collide*,
- but red points are not allowed to collide with blue points.

For example:



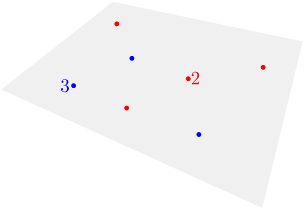
$\in \mathcal{M}_5$

Magnetic monopole moduli spaces

Theorem (Donaldson 1984):
The magnetic monopole moduli space $\mathcal{M}_n$ is a configuration space.
Precisely, it is the configuration space of:

- n red points in $\mathbf{R}^2$ *that are allowed to collide*,
- n blue points in $\mathbf{R}^2$ *that are allowed to collide*,
- but red points are not allowed to collide with blue points.

For example:



$\in \mathcal{M}_5$

Note: *In order to get a configuration space model in* $\mathbf{R}^2$*, Donaldson's theorem breaks the symmetry of* $\mathbf{R}^3$ *by choosing a way of splitting it into* $\mathbf{R}^2$ *and* $\mathbf{R}$*.*

Configuration spaces and their applications

Outline

Configuration spaces

(Points)

(Discs)

Braid groups and low-dim. topology

(Alexander/Markov)

Hurwitz spaces and number theory

Magnetic monopole moduli spaces

(Donaldson)

(Kottke-Singer)

Summary

Magnetic monopole moduli spaces

A consequence of Donaldson's theorem:

Magnetic monopole moduli spaces

A consequence of Donaldson's theorem:

Theorem (Cohen, Cohen, Mann, Milgram 1991):
$H_d(\mathcal{M}_n) \cong H_d(C_{2n}(\mathbf{R}^2))$ for every $n \geqslant 1$ and $d \geqslant 0$
The magnetic monopole moduli space $\mathcal{M}_n$ and the configuration space $C_{2n}(\mathbf{R}^2)$ have the same number of d-dimensional holes, for each d.

Configuration spaces and their applications

Outline

Configuration spaces
(Points)
(Discs)

Braid groups and low-dim. topology
(Alexander/Markov)

Hurwitz spaces and number theory

Magnetic monopole moduli spaces
(Donaldson)
(Kottke-Singer)

Summary

Magnetic monopole moduli spaces

A consequence of Donaldson's theorem:

Theorem (Cohen, Cohen, Mann, Milgram 1991):
$H_d(\mathcal{M}_n) \cong H_d(C_{2n}(\mathbf{R}^2))$ for every $n \geqslant 1$ and $d \geqslant 0$

The magnetic monopole moduli space $\mathcal{M}_n$ and the configuration space $C_{2n}(\mathbf{R}^2)$ have the same number of d-dimensional holes, for each d.

More recently:

The space $\mathcal{M}_n$ is non-compact — can it be compactified?

Magnetic monopole moduli spaces

A consequence of Donaldson's theorem:

Theorem (Cohen, Cohen, Mann, Milgram 1991):
$H_d(\mathcal{M}_n) \cong H_d(C_{2n}(\mathbf{R}^2))$ for every $n \geqslant 1$ and $d \geqslant 0$
The magnetic monopole moduli space $\mathcal{M}_n$ and the configuration space $C_{2n}(\mathbf{R}^2)$ have the same number of d-dimensional holes, for each d.

More recently:

The space $\mathcal{M}_n$ is non-compact — can it be compactified?

Theorem (Kottke, Singer 2022):
$\mathcal{M}_n$ can be (partially) compactified using configuration spaces.

Magnetic monopole moduli spaces

A consequence of Donaldson's theorem:

Theorem (Cohen, Cohen, Mann, Milgram 1991):
$H_d(\mathcal{M}_n) \cong H_d(C_{2n}(\mathbf{R}^2))$ for every $n \geqslant 1$ and $d \geqslant 0$
The magnetic monopole moduli space $\mathcal{M}_n$ and the configuration space $C_{2n}(\mathbf{R}^2)$ have the same number of d-dimensional holes, for each d.

More recently:

The space $\mathcal{M}_n$ is non-compact — can it be compactified?

Theorem (Kottke, Singer 2022):
$\mathcal{M}_n$ can be (partially) compactified using configuration spaces.
Precisely, the compactification consists of configurations of:

- $k \leqslant n$ points in $\mathbf{R}^3$ *that may not collide*,
- an additional "non-local" parameter in $\mathcal{M}_{c_i}$ for each point
- $c_1 + \cdots + c_k = n$

Summary

Summary

- Configurations of point-particles in Euclidean space: $C_n(\mathbf{R}^d)$
- For configurations of *discs*, there are phase changes as the discs' radius changes.
- $C_n(\mathbf{R}^2) \rightsquigarrow$ braid groups $\mathbf{B}_n \rightsquigarrow$ knots and links (Markov moves)
- Enrichment of $C_n(\mathbf{R}^2) \rightsquigarrow$ Hurwitz spaces $\mathrm{Hur}^c_{G,n} \rightsquigarrow$ related to number theoretical questions as $n \to \infty$.
- Magnetic monopole moduli spaces $\rightsquigarrow$ configuration models $\rightsquigarrow$ partial compactifications by configuration space models

Summary

Summary

- Configurations of point-particles in Euclidean space: $C_n(\mathbf{R}^d)$
- For configurations of *discs*, there are phase changes as the discs' radius changes.
- $C_n(\mathbf{R}^2) \rightsquigarrow$ braid groups $\mathbf{B}_n \rightsquigarrow$ knots and links (Markov moves)
- Enrichment of $C_n(\mathbf{R}^2) \rightsquigarrow$ Hurwitz spaces $\mathrm{Hur}^c_{G,n} \rightsquigarrow$ related to number theoretical questions as $n \to \infty$.
- Magnetic monopole moduli spaces $\rightsquigarrow$ configuration models $\rightsquigarrow$ partial compactifications by configuration space models

Thank you for listening!

References

- J. Alexander, *A lemma on a system of knotted curves*, Proceedings of the National Academy of Sciences of the United States of America (1923)
- H. Alpert, M. Kahle, R. MacPherson, *Configuration spaces of disks in an infinite strip*, Journal of Applied and Computational Topology (2021)
- M. Atiyah, N. Hitchin, *The geometry and dynamics of magnetic monopoles*, Princeton University Press (1988)
- G. Carlsson, J. Gorham, M. Kahle, J. Mason, *Computational topology for configuration spaces of hard disks*, Physical Review E (2012)
- F. Cohen, R. Cohen, B. Mann, R. Milgram, *The topology of rational functions and divisors of surfaces*, Acta Mathematica (1991)
- P. Dirac, *Quantised singularities in the electromagnetic field*, Proceedings of the Royal Society A (1931)
- S. Donaldson, *Nahm's equations and the classification of monopoles*, Communications in Mathematical Physics (1984)
- J. Ellenberg, A. Venkatesh, C. Westerland, *Homological stability for Hurwitz spaces and the Cohen-Lenstra conjecture over function fields*, Annals of Mathematics (2016)
- C. Kottke, M. Singer, *Partial compactification of monopoles and metric asymptotics*, Memoirs of the American Mathematical Society (2022)
- A. Markov, *Über die freie Äquivalenz der geschlossenen Zöpfe*, Matematicheskiĭ Sbornik (1936)

Image credits: robot arm - magnetic monopole image - 3-strand braid - table of knots