Are there non-zero compactly-supported homology classes on mapping class groups of infinite-type surfaces? Part II

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Abstract.

The Mumford conjecture – a consequence of the Madsen-Weiss theorem – describes the rational homology of the mapping class groups $\mathrm{Mod}(\Sigma_{g,1})$ in the limit as g goes to infinity, in terms of the dual *Miller-Morita-Mumford classes* (MMM classes). Instead of the colimit of the mapping class groups, one may instead take the colimit of the surfaces $\Sigma_{g,1}$ themselves, to obtain an infinite-type surface Σ_∞, and consider its mapping class group $\mathrm{Mod}(\Sigma_\infty)$, called the "big mapping class group". The structure of its homology is very mysterious, and very large: it is uncountably generated in every positive degree. There is a natural injective homomorphism

$$\operatorname*{colim}_{g\to\infty}(\mathrm{Mod}(\Sigma_{g,1})) \longrightarrow \mathrm{Mod}(\Sigma_\infty),$$

and it is natural to ask what its effect on homology is – in particular, do the dual MMM classes vanish on $\mathrm{Mod}(\Sigma_\infty)$? This is a special case of a more general question – for any infinite-type surface S, one may ask whether its mapping class group $\mathrm{Mod}(S)$ admits non-zero homology classes supported on a compact subsurface of S. We will give a complete answer to this question when S has non-zero genus (including the case $S = \Sigma_\infty$) and a partial answer when S has genus zero.

This talk follows on from an earlier talk with the same title in January 2024, where some of these results were presented modulo a gap in the proof. The gap has now been filled with the help of a 2-dimensional analogue of an "infinite iteration trick" used by Mather, Berrick and others, which I will explain in the talk.

This represents joint work with Xiaolei Wu and is based on arXiv:2405.03512.