Do the dual Miller-Morita-Mumford classes vanish in the homology of the big mapping class group?

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Abstract.

The Mumford conjecture – a consequence of the Madsen-Weiss theorem – describes the rational homology of the mapping class groups $\mathrm{Mod}(\Sigma_{g,1})$ in the limit as g goes to infinity, in terms of the dual Miller-Morita-Mumford (MMM) classes. Instead of taking the colimit of the mapping class groups, one may instead take the colimit of the surfaces $\Sigma_{g,1}$ themselves, to obtain an infinite-type surface Σ_∞, and consider its mapping class group $\mathrm{Mod}(\Sigma_\infty)$, called the "big mapping class group". The structure of its homology is very mysterious, and very large: it is uncountably generated in every positive degree. There is a natural homomorphism from the colimit of $\mathrm{Mod}(\Sigma_{g,1})$ to $\mathrm{Mod}(\Sigma_\infty)$, and one may wonder what its effect is on homology; in particular whether the dual MMM classes vanish on $\mathrm{Mod}(\Sigma_\infty)$. This is a special case of a more general question for any infinite-type surface S: does its mapping class group $\mathrm{Mod}(S)$ admit non-zero homology classes supported on a compact subsurface of S? We will give a complete answer to this question when S has non-zero genus (including the case $S = \Sigma_\infty$) and a partial answer when S has genus zero. This represents joint work with Xiaolei Wu and is based on arXiv:2405.03512.