Compactly-supported homology classes for big mapping class groups

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Abstract.

The Mumford conjecture – a consequence of the Madsen-Weiss theorem – describes the (rational) homology of the colimit of the mapping class groups $\mathrm{Mod}(\Sigma_{g,1})$ as g goes to infinity. One may alternatively take the colimit of the surfaces $\Sigma_{g,1}$ themselves, to obtain an infinite-type surface Σ_∞ and then consider the homology of its mapping class group $\mathrm{Mod}(\Sigma_\infty)$, which is uncountably generated in all positive degrees and whose precise structure is very mysterious. There is a natural homomorphism from the former to the latter, and it is a natural question to ask whether its image is non-zero.

One may more generally ask, for any infinite-type surface S, whether $\mathrm{Mod}(S)$ admits non-zero homology classes supported on a compact subsurface of S. We will give a complete answer to this question when S has positive (possibly infinite) genus and a partial answer when S has genus zero. This represents joint work with Xiaolei Wu.