Homology of big MCGs

Setting

S = *infinite-type* surface
$\mathrm{Mod}(S) = \mathrm{Homeo}(S)/\mathrm{iso}$

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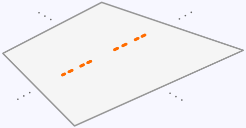
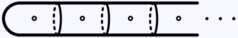
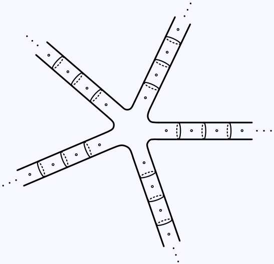
$S_{\mathcal{C}} =$ 

$= \mathbb{R}^2 \smallsetminus$ Cantor

$F_1 =$ 

$F_n =$ 

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Questions

(1) $H_*(\mathrm{Mod}(S)) \cong ?$

(2) What is the map

$$\underset{\substack{\Sigma \subset S \\ compact}}{\mathrm{colim}} \big(H_*(\mathrm{Mod}(\Sigma))\big) \xrightarrow{(*)} H_*(\mathrm{Mod}(S)) \quad ?$$

(What is its image?)

(Is it zero?)

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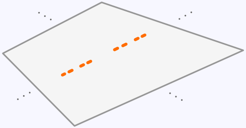
$S_{\mathcal{C}} =$ 

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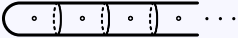
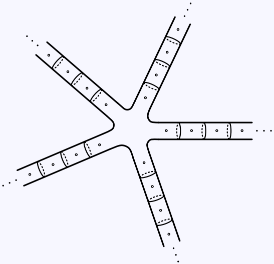
$F_1 =$ 

$F_n =$ 

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Remark

Domain of $(*)$ is understood by [Madsen-Weiss, 2007] in the case $\mathrm{genus}(S) = \infty$.

Rationally $\cong \mathbb{Q}[\kappa_1, \kappa_2, \dots]^*$

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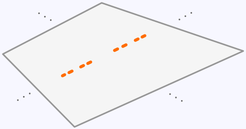
$S_{\mathcal{C}} =$ 

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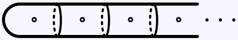
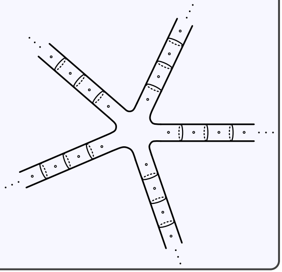
$F_1 =$ 

$F_n =$ 

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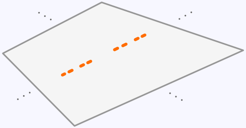
$S_{\mathcal{C}} =$ 

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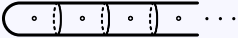
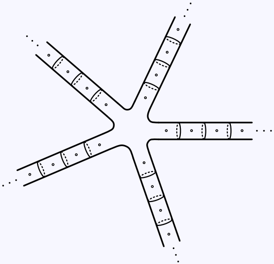
$F_1 =$ 

$F_n =$ 

Answers

Theorem [P.-Wu, 2022-24]

	(1)	**(2)**
S_∞	$\supseteq \mathbb{Z}^{\oplus \mathfrak{c}}\ \forall *$	0
$S_{\mathcal{C}}$	$\begin{cases} \mathbb{Z} & * \text{ even} \\ 0 & * \text{ odd} \end{cases}$	0
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$(n \geqslant 4)$

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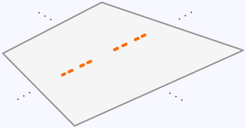
$S_{\mathcal{C}} =$ 

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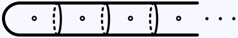
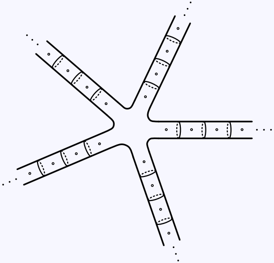
$F_1 =$ 

$F_n =$ 

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Proofs – key ideas

Homological stability

Pseudo-mitotic/binate grps

Top. distinguished ends

Escaping sequences of SCCs

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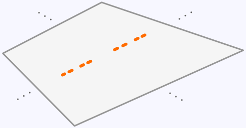

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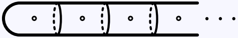

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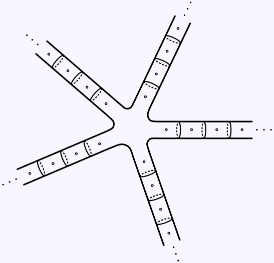

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Open questions

(A) $\exists$ surface S: $H_*(\mathrm{Mod}(S)) \supseteq A$, *A torsion* & uncountable?

(B) Is there a dichotomy

- $\forall i$, $H_i(\mathrm{Mod}(S))$ fin. gen.
- $\forall i$, $|H_i(\mathrm{Mod}(S))| > \aleph_0$?