

Homological stability for asymptotic monopole moduli spaces

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Abstract.

Magnetic monopoles were introduced by Dirac in 1931 to explain the quantisation of electric charges. In his model, they are singular solutions to an extension of Maxwell's equations allowing non-zero magnetic charges. An alternative model, developed by 't Hooft and Polyakov in the 1970s, is given, after a certain simplification, by smooth solutions to a different set of equations, the *Bogomolny equations*, whose moduli space of solutions has connected components M_k indexed by positive integers k (the "total magnetic charge"). These moduli spaces, which are non-compact manifolds, have an interpretation in terms of rational self-maps of $\mathbb{C}\mathbb{P}^1$ due to Donaldson and their stable homotopy types may be described in terms of braid groups by a result of F. Cohen, R. Cohen, Mann and Milgram. A partial compactification of M_k has recently been constructed by Kottke and Singer, whose boundary strata may be called "ideal" or "asymptotic" monopole moduli spaces. I will describe joint work with Ulrike Tillmann in which we prove the existence of stability patterns in the homology of these spaces.