Introduction

The overarching theme of my research is to try to understand the topology of different kinds of *moduli spaces*, through their homology and their fundamental groups. The kinds of moduli spaces that I have studied include configuration spaces, often with additional non-local structure (an alternating structure; configuration-section spaces; asymptotic monopole moduli spaces), moduli spaces of disconnected submanifolds and mapping class groups of surfaces, including surfaces of infinite type.

On the homological side, I have proven homological stability results for many of these moduli spaces, as well as more subtle periodicity results in settings where homological stability fails. On the group-theoretic side, I have studied the lower central series of surface, virtual and welded braid groups, and constructed new (functorial) homological representations of motion groups and mapping class groups.

Moduli spaces

The concept of a *moduli space* is of central importance in many areas of mathematics, parametrising collections of all mathematical (or physical) objects of a given kind. The broad goal of my research so far has been to understand the topology of different kinds of moduli spaces, principally via their homology (§1) and via their fundamental groups (§2). The kinds of moduli spaces that I have studied include:

1. Configuration spaces of points in manifolds, especially *non-local configuration spaces*, in which the configurations are equipped with some kind of additional "non-local" structure, such as:
   (a) An ordering modulo even permutations (alternating structure) [Pal13; MP15a; MP15b].
   (b) A "field" defined on the complement of the configuration [PT21; PT22]. These moduli spaces generalise *Hurwitz spaces*, which correspond to the setting where the underlying manifold is $\mathbb{C}$ and the fields take values in BG for a discrete group G.
   (c) Non-local data encoding the interactions of *asymptotic magnetic monopoles* [PT23].
2. The homology of configuration spaces with respect to *polynomial twisted coefficient systems* [Pal18].
3. The *motivic cohomology* of configuration schemes on a given smooth scheme [HP23].
4. Moduli spaces of higher-dimensional disconnected submanifolds [Pal21]. These are related to moduli spaces of manifolds with Baas-Sullivan singularities.
5. Mapping class groups of surfaces, including surfaces of *infinite type* [PW24a; PW24b; PW25a].
6. Families of "Thompson-like" groups (labelled Thompson groups and twisted Brin-Thompson groups) [PW25b].

1. Homology of moduli spaces

An important tool to understand the homology of moduli spaces is the phenomenon of *homological stability*. For a family of moduli spaces indexed by a parameter n, it means that their homology is independent of n in an unbounded range of degrees as $n \to \infty$. An important example of this is for the mapping class groups of compact, connected, orientable surfaces: in this setting, homological stability was proven by Harer [Har85] and the limiting homology was computed by Madsen and Weiss [MW07]; these results together settled the *Mumford conjecture* [Mum83]. Homological stability also has applications to analytic number theory. For example, Ellenberg, Venkatesh and Westerland [EVW16] proved an asymptotic version of the *Cohen-Lenstra conjecture* for function fields, the core of their proof being a homological stability result for Hurwitz spaces. More recently, Bergström, Diaconu, Petersen and Westerland [BDPW23] reduced the proof of another conjecture in analytic number theory — asymptotic formulas for the moments of the central values of families of quadratic L-functions conjectured by [CFKRS05] — to a certain twisted homological stability statement for the braid groups, a proof of which was subsequently found by Miller, Patzt, Petersen and Randal-Williams [MPPR24].

A significant part of my research has been concerned with (or closely related to) establishing homological stability for different kinds of moduli spaces, including each of the kinds of moduli spaces from points 1.–4. in the list above. There are also settings where homological stability is false and instead there exist more complicated periodic patterns in the homology in a stable range of degrees. This holds for example for configuration spaces on *closed* manifolds [CP15].

In the context of mapping class groups of infinite-type surfaces (also known as "big mapping class groups"; point 5. above), I have used homological stability techniques to prove that certain families of big mapping class groups have trivial homology in all degrees (i.e. they are acyclic) [PW24b]. In contrast with this, another result [PW24a] shows that many other families of big mapping class groups have *uncountable* (integral) homology in all degrees. In a similar vein, I have also worked on proving acyclicity for different families of Thompson-like groups (namely for *labelled Thompson groups* and *twisted Brin-Thompson groups*; point 6. above) [PW25b]. These families of groups are key examples in geometric group theory and the acyclicity of these groups has important group-theoretical corollaries.

2. Fundamental groups of moduli spaces

The key examples of fundamental groups of moduli spaces that I am interested in are *motion groups* (including surface braid groups and loop braid groups) and *mapping class groups* of surfaces, which have far-reaching connections to knot theory and physics (among many other areas). Two lenses through which one can study these groups are through their *lower central series* and through their *representation theory*.

Understanding the *lower central series* of a group (and its associated Lie algebra) is typically a difficult task, but it can lead to a deep understanding of the underlying structure of the group. A fundamental problem when studying the *representation theory* of a group is to determine whether it admits any faithful representation on a finite-dimensional vector space – in other words, whether it is *linear*. This is known to be true for the classical braid groups [Big01; Kra02] but is wide open for almost all other motion groups and mapping class groups.

Motivated by these problems, my previous work in this direction has been concerned with:

- Studying the *lower central series* of surface braid groups, loop braid groups and virtual braid groups, as well as *partitioned* versions of all of these groups, interpolating between the ordered and unordered versions [DPS25]. In particular, we answered the question of whether the lower central series *stops* (i.e. stabilises after finitely many steps) for every one of these groups (with a single family of exceptions involving partitioned braid groups on the projective plane). The answer turns out to depend subtly on the number of strands, how they are partitioned and the topology of the underlying surface.
- New topological constructions of representations of (surface) braid groups, loop braid groups and mapping class groups, involving:
  - non-abelian (Heisenberg) local systems for representations of mapping class groups [BPS25],
  - functorial representations of categories of manifolds and embeddings [PS24],
  - polynomiality results for the above functorial representations [PS25a],
  - pro-nilpotent towers of homological representations [PS25b], including a pro-nilpotent tower that extends the Lawrence-Krammer-Bigelow representations.

References

[BDPW23] J. Bergström, A. Diaconu, D. Petersen and C. Westerland. *Hyperelliptic curves, the scanning map, and moments of families of quadratic L-functions*. ArXiv:2302.07664. 2023.

[Big01] S. J. Bigelow. *Braid groups are linear*. *J. Amer. Math. Soc.* 14.2 (2001), 471–486 (electronic).

[BPS25] C. Blanchet, M. Palmer and A. Shaukat. *Heisenberg homology on surface configurations*. *Math. Ann.* 393 (2025), pp. 1989–2056.

[CP15] F. Cantero and M. Palmer. *On homological stability for configuration spaces on closed background manifolds*. *Doc. Math.* 20 (2015), pp. 753–805.

[CFKRS05] J. B. Conrey, D. W. Farmer, J. P. Keating, M. O. Rubinstein and N. C. Snaith. *Integral moments of L-functions*. *Proc. London Math. Soc. (3)* 91.1 (2005), pp. 33–104.

[DPS25] J. Darné, M. Palmer and A. Soulié. *When the lower central series stops: a comprehensive study for braid groups and their relatives*. *Mem. Amer. Math. Soc.* 309.1563 (2025), pp. v+124.

[EVW16] J. S. Ellenberg, A. Venkatesh and C. Westerland. *Homological stability for Hurwitz spaces and the Cohen-Lenstra conjecture over function fields*. *Ann. of Math. (2)* 183.3 (2016), pp. 729–786.

[Har85] J. L. Harer. *Stability of the homology of the mapping class groups of orientable surfaces*. *Ann. of Math. (2)* 121.2 (1985), pp. 215–249.

[HP23] G. Horel and M. Palmer. *Motivic homological stability of configuration spaces*. *Bull. Lond. Math. Soc.* 55.2 (2023), pp. 892–913.

[Kra02] D. Krammer. *Braid groups are linear*. *Ann. of Math. (2)* 155.1 (2002), pp. 131–156.

[MW07] I. Madsen and M. Weiss. *The stable moduli space of Riemann surfaces: Mumford's conjecture*. *Ann. of Math. (2)* 165.3 (2007), pp. 843–941.

[MP15a] J. Miller and M. Palmer. *A twisted homology fibration criterion and the twisted group-completion theorem*. *Q. J. Math.* 66.1 (2015), pp. 265–284.

[MP15b] J. Miller and M. Palmer. *Scanning for oriented configuration spaces*. *Homology, Homotopy and Applications* 17.1 (2015), pp. 35–66.

[MPPR24] J. Miller, P. Patzt, D. Petersen and O. Randal-Williams. *Uniform twisted homological stability*. ArXiv:2402.00354. 2024.

[Mum83] D. Mumford. *Towards an enumerative geometry of the moduli space of curves*. *Arithmetic and geometry, Vol. II*. Vol. 36. Progr. Math. Boston, MA: Birkhäuser Boston, 1983, pp. 271–328.

[Pal13] M. Palmer. *Homological stability for oriented configuration spaces*. *Trans. Amer. Math. Soc.* 365.7 (2013), pp. 3675–3711.

[Pal18] M. Palmer. *Twisted homological stability for configuration spaces*. *Homology, Homotopy and Applications* 20.2 (2018), pp. 145–178.

[Pal21] M. Palmer. *Homological stability for moduli spaces of disconnected submanifolds, I*. *Algebr. Geom. Topol.* 21.3 (2021), pp. 1371–1444.

[PS24] M. Palmer and A. Soulié. *Topological representations of motion groups and mapping class groups – a unified functorial construction*. *Annales Henri Lebesgue* 7 (2024), pp. 409–519.

[PS25a] M. Palmer and A. Soulié. *Polynomiality of surface braid and mapping class group representations*. *Trans. Amer. Math. Soc.* 378.8 (2025), pp. 5769–5843.

[PS25b] M. Palmer and A. Soulié. *The pro-nilpotent Lawrence-Krammer-Bigelow representation*. *J. Pure Appl. Algebra* 229.6 (2025), Paper No. 107952.

[PT21] M. Palmer and U. Tillmann. *Configuration-mapping spaces and homology stability*. *Res. Math. Sci.* 8.3 (2021), Paper No. 38.

[PT22] M. Palmer and U. Tillmann. *Point-pushing actions for manifolds with boundary*. *Groups Geom. Dyn.* 16.4 (2022), pp. 1179–1224.

[PT23] M. Palmer and U. Tillmann. *Homology stability for asymptotic monopole moduli spaces*. *Proc. R. Soc. A* 479, 20230300 (2023).

[PW24a] M. Palmer and X. Wu. *Big mapping class groups with uncountable integral homology*. *Doc. Math.* 29.1 (2024), pp. 159–189.

[PW24b] M. Palmer and X. Wu. *On the homology of big mapping class groups*. *Journal of Topology* 17.4 (2024), Paper no. e12358.

[PW25a] M. Palmer and X. Wu. *Compact and finite-type support in the homology of big mapping class groups*. *J. Lond. Math. Soc. (2)* 112.3 (2025), Paper No. e70258.

[PW25b] M. Palmer and X. Wu. *Embedding groups into acyclic groups*. ArXiv:2510.16879. 2025.